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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, February 2015
(2013 Scheme)
Branch : Mechanical Engineering
Streams : Thermal Engineering and Propulsion Engineering
MMA 1001 : APPLIED MATHEMATICS

Time : 3 Hours Max. Marks : 60

***Instruction :** Answer **any two** questions from **each** Module. **All** questions carry **equal** marks.*

Module – I

1. a) Prove that $J_n'(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$

   b) Prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$ where α and β are distinct roots of $J_n(x) = 0$.

2. a) Show that $(n + 1)\, P_{n+1}(x) = (2n + 1)\, P_n(x) - nP_{n-1}(x)$.

   b) Express the polynomial $x^3 - 3x^2 + 4x - 6$ in terms of Legendre polynomials.

3. Solve the boundary value problem $u_t = u_{xx}$ given that

   $u(\pi/2, t) = 0$, $u_t(0, t) = 0$; $u(x, 0) = 30 \cos 5x$, using Laplace Transform method.

Module – II

4. a) State and prove a necessary condition for $\int_a^b f(x, y, y')\, dx$ to be an extremum.

   b) Prove that the extremal of the isoperimetric problem $v[y(x)] = \int_1^4 (y')^2\, dx$, $y(1) = 3$, $y(4) = 24$ subject to the condition $\int_1^4 y\, dx = 36$ is a parabola.

P.T.O.

5. a) Find the integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 5\sin x \text{ with initial condition } y(0) = 1 \text{ and } y'(0) = -2.$$

b) Solve the integral equation $y(x) = x + 2\int_0^x \cos(x-t)\,y(t)\,dt$ by convolution method.

6. a) Define DFT and Inverse DFT. Find the Inverse DFT of (2, 4 + 4i, – 6, 4 – 4i).

b) Find the DFT of (2, 4 + 4i, – 6, 4 – 4i) using the FFT algorithm.

Module – III

7. a) Define subspace of a vector space. Give three subspaces of R^3.

b) Define basis and dimension of a vector space. Find the dimension of the null space and column space of $\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.

8. a) $T : R^3 \to R^3$ is defined by T (x, y, z) = (x + 2y – z, y + z, x + y – 2z). Show that T is linear. Also find the matrix of T relative to the basis. $v_1 = (1, -1, 1)$; $v_2 = (1, 0, -2)$; $v_3 = (1, -2, 1)$.

b) Show that a linear transformation $T : U \to V$ is one to one if and only if Kernal (T) = {0}.

9. Use Gram-Schmidt process, find an orthonormal basis of the subspace of R^3 spanned by the vectors $\left\{ v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$